Revnet Value Flows as a Continuous–Time Dynamical System: A Rate–Based Formalization

The CEL Team

November 10, 2025

Abstract

We formalize the economics of a Revnet as a continuous–time dynamical system driven by expected rates of cash–ins (issuance) and cash–outs (redemption). Within each stage of the protocol, the issuance price is predetermined and evolves by discrete, scheduled steps; between those steps, the state follows a system of ordinary differential equations (ODEs). We derive the core balance equations for the treasury and token supply, characterize the time evolution of the marginal redemption floor, obtain closed–form solutions under constant rates within a stage, and identify steady states and neutral lines for floor dynamics. The analysis is self–contained and uses only issuance and redemption flows in expectation; an optional appendix shows how loan flows can be incorporated as rate terms without changing the qualitative conclusions.

Contents

1 Introduction

A *Revnet* is an autonomous, tokenized financial structure with immutable issuance and redemption rules enforced by smart contracts. At any time, external users may pay in a base asset to mint new tokens at a stage–specified issuance price; token holders may redeem tokens for a programmatic share of the treasury through a convex bonding curve featuring a *cash–out tax*. While individual interactions are discrete on–chain events, it is often analytically and empirically useful to study their aggregate effect in continuous time, replacing random and user–driven flows by *expected rates*.

This document develops a rate–based, continuous–time formalization. It expresses the value flows as ODEs, establishes invariants and monotonicity properties, and provides closed–form solutions on time windows where the issuance price is constant. The presentation is designed to be mathematically robust yet directly usable for simulation, estimation, and stage design.

2 Model Setup

2.1 Time, stages, and issuance price

Let time be continuous, $t \in [0, \infty)$. The protocol operates in a sequence of stages indexed by $k \in \mathbb{N}$. Each stage k starts at time $t_k \in [0, \infty)$ with $t_1 < t_2 < \cdots$, and is characterized by a tuple of immutable parameters

$$\mathcal{S}_k = \big(t_k,\, P_{\text{issue},k,0},\, \gamma_{\text{cut},k},\, \Delta t_k,\, \sigma_k,\, r_k,\, \mathcal{A}_k\big).$$

Within stage k, the *contract issuance price* $P_{\text{issue},k}(\cdot)$ is a right–continuous step function defined by

$$P_{\text{issue},k}(t) = P_{\text{issue},k,0}\, \gamma_k^{\lfloor (t-t_k)/\Delta t_k \rfloor}, \qquad \gamma_k := \frac{1}{1-\gamma_{\text{cut},k}} \in (1, \infty), \tag{1}$$

for $t \in [t_k, t_{k+1})$, where t_{k+1} is the start time of the next stage (with the convention $t_{K+1} = \infty$ for a last stage K). The per–mint split parameter is $\sigma_k \in [0, 1)$; the cash–out tax is $r_k \in [0, 1)$; and $\mathcal{A}_k$ is a finite list of auto–issuances, each specified by a pair (τ, a) with $\tau \in [t_k, t_{k+1})$ and $a > 0$ tokens to mint at τ.

2.2 State variables and rate processes

At each time $t \geq 0$, the economic state is summarized by the pair $(B(t), S(t))$, where $B(t) \in \mathbb{R}_{\geq 0}$ denotes the treasury balance in base units and $S(t) \in \mathbb{R}_{\geq 0}$ denotes the circulating token supply. The initial condition $(B(0), S(0))$ is given with $S(0) > 0$.

We describe the aggregate user activity by two nonnegative measurable rate functions:

- the *cash–in rate* $r_{\text{in}} : [0, \infty) \to \mathbb{R}_{\geq 0}$, with units of base asset per unit time, representing expected pay–ins routed to issuance; and
- the *cash–out rate* $r_{\text{out}} : [0, \infty) \to \mathbb{R}_{\geq 0}$, with units of tokens per unit time, representing expected redemptions through the bonding curve. This rate is defined so that S decreases at instantaneous rate r_{out} due to redemptions, inclusive of any protocol token fees that are burned at redemption time.

We assume $r_{\text{in}}, r_{\text{out}} \in L^1_{\text{loc}}([0, \infty))$. Let $\phi_{\text{tot}} \in [0, 1)$ denote the aggregate token fee fraction applied at redemption before the user–side curve (e.g., a split into two external protocol fees summing to ϕ_{tot}). We do not require r_{in} and r_{out} to be constant.

2.3 Redemption curve and marginal prices

Let k be the active stage at time t, and let $(S, B) \in (0, \infty)^2$. If a holder redeems an amount $q \in [0, S]$ of tokens instantaneously, the (pre–fee) bonding curve pays out

$$C_k(q; S, B) = \frac{q}{S}\, B \left[(1 - r_k) + r_k \frac{q}{S}\right]. \tag{2}$$

The marginal redemption price at $q = 0$ equals

$$\frac{\partial}{\partial q} C_k(q; S, B)\Big|_{q=0} = (1 - r_k)\frac{B}{S}. \tag{3}$$

The user–facing marginal floor at time t is therefore

$$P^{\text{floor}}(t) = (1 - \phi_{\text{tot}})(1 - r_k)\frac{B(t)}{S(t)}. \tag{4}$$

3 Core Dynamics with Cash–Ins and Cash–Outs in Expectation

3.1 ODEs between issuance–price steps

Fix a stage k and a subinterval $I \subseteq [t_k, t_{k+1})$ on which the issuance price is constant, $P_{\text{issue}}(t) \equiv P_{\text{issue},k}(t) =: \bar{P} > 0$, and no auto–issuance occurs. On such an interval, the expected dynamics of the state (B, S) are governed by the ODE system

$$\dot{S}(t) = \frac{r_{\text{in}}(t)}{\bar{P}} \; - \; r_{\text{out}}(t), \tag{5}$$

$$\dot{B}(t) = r_{\text{in}}(t) \; - \; (1 - r_k)\frac{B(t)}{S(t)}\, r_{\text{out}}(t). \tag{6}$$

Equation (5) states that issuance mints tokens at rate $r_{\text{in}}/\bar{P}$ while redemptions burn tokens at rate r_{out}. Equation (6) states that the treasury increases at rate r_{in} from pay–ins and decreases at the *marginal* redemption drain $(1 - r_k)(B/S)r_{\text{out}}$ implied by (3). The token fee parameter ϕ_{tot} does not appear in (6) because, to first order in q, the user leg and each protocol–fee leg together burn r_{out} tokens while the treasury outflow is entirely determined by the marginal curve (3).

3.2 Jump conditions at issuance–price steps and auto–issuances

At scheduled issuance–price cut times inside a stage, $t = \tau \in (t_k, t_{k+1})$ with $\tau - t_k = m\Delta t_k$ for some $m \in \mathbb{N}$, equations (5)–(6) remain valid but the constant $\bar{P}$ changes discontinuously to the next step level as prescribed by (1). The state (B, S) is continuous across such times.

If an auto–issuance of $a > 0$ tokens is scheduled at some $\tau \in (t_k, t_{k+1})$, then S jumps by a at τ while B is unchanged:

$$S(\tau^+) = S(\tau^-) + a, \qquad B(\tau^+) = B(\tau^-). \tag{7}$$

Equations (5)–(6) hold on the open subintervals determined by such impulses.

3.3 Existence, uniqueness, and state constraints

Proposition 3.1 (Well–posedness)**.** *Fix a stage k and an interval $I \subseteq [t_k, t_{k+1})$ with constant issuance price $\bar{P} > 0$ and no auto–issuance. Suppose $r_{\text{in}}, r_{\text{out}} \in L^1_{\text{loc}}(I)$ and $(B(t_0), S(t_0)) \in (0, \infty)^2$ at some $t_0 \in I$. Then the system (5)–(6) admits a unique absolutely continuous solution $(B, S) : I \to (0, \infty)^2$. Moreover, if S is driven towards zero, the maximal interval of existence ends at the first time τ with $\lim_{t \uparrow \tau} S(t) = 0$.*

Proof. The right–hand sides of (5)–(6) are Carathéodory functions on $(0,\infty)^2$, locally integrable in time and locally Lipschitz in (B,S) away from the boundary $\{S=0\}$. Standard ODE theory ensures well–posedness on any compact subinterval contained in $\{S>0\}$. Positivity is preserved because (6) implies $\dot{B}(t) \geq -(1-r_k)(B/S)r_{\text{out}}$, so B cannot cross zero from above, and (5) implies that S decreases at finite speed. □

In applications, one enforces the viability constraint that the integrated redemption outflow never exceeds the available supply, ensuring $S(t) > 0$ for all t.

4 Floor Dynamics and Consequences

4.1 Time derivative of the marginal floor

Differentiating (4) and substituting (5)–(6) yields

$$\dot{P}^{\text{floor}}(t) = (1-\phi_{\text{tot}})(1-r_k)\,\frac{S(t)\,\dot{B}(t) - B(t)\,\dot{S}(t)}{S(t)^2} = (1-\phi_{\text{tot}})(1-r_k)\,\frac{r_{\text{in}}(t)\Big(S(t) - \frac{B(t)}{\bar{P}}\Big) + r_k\,B(t)\,r_{\text{out}}(t)}{S(t)^2}. \tag{8}$$

Equation (8) makes transparent two distinct sources of floor appreciation: the issuance term is strictly positive whenever the contract issuance price $\bar{P}$ exceeds the current backing per token B/S; the redemption term is strictly positive for all $r_k > 0$ and $r_{\text{out}}(t) > 0$.

Proposition 4.1 (Issuance threshold and redemption monotonicity)**.** *Fix a stage and a subinterval with constant $\bar{P} > 0$ and no auto–issuance. Then:*

1. *If $r_{\text{out}} \equiv 0$, the floor increases, $\dot{P}^{\text{floor}}(t) > 0$, if and only if $\bar{P} > B(t)/S(t)$.*
2. *If $r_{\text{in}} \equiv 0$ and $r_k > 0$, the floor strictly increases for any positive outflow, $\dot{P}^{\text{floor}}(t) > 0$ whenever $r_{\text{out}}(t) > 0$.*

Proof. Both statements are immediate from (8). □

4.2 Neutral line and comparative statics

For fixed $\bar{P}$, the locus in $(r_{\text{in}}, r_{\text{out}})$–space where the floor is locally stationary, $\dot{P}^{\text{floor}}(t) = 0$, is the straight line

$$r_{\text{in}}(t)\left[S(t) - \frac{B(t)}{\bar{P}}\right] + r_k\,B(t)\,r_{\text{out}}(t) = 0. \tag{9}$$

If $S(t) > \frac{B(t)}{\bar{P}}$, points above this line (larger r_{in} and/or larger r_{out}) produce floor growth; if $S(t) < \frac{B(t)}{\bar{P}}$, issuance must be sufficiently strong to offset the negative contribution of the first term.

At steady state (treated next), the floor pins to a simple multiple of the issuance price; that identity gives an immediate handle for stage design.

5 Closed–Form Solutions and Steady States under Constant Rates

Consider a time window $I = [t_0, t_1) \subseteq [t_k, t_{k+1})$ with constant issuance price $\bar{P} > 0$, no auto–issuance, and constant rates $r_{\text{in}} \geq 0$, $r_{\text{out}} \geq 0$. Define the net supply drift

$$\beta := \frac{r_{\text{in}}}{\bar{P}} - r_{\text{out}} \in \mathbb{R}.$$

Then S evolves linearly,

$$S(t) = S_0 + \beta\,(t - t_0), \qquad S_0 := S(t_0). \tag{10}$$

The treasury satisfies a linear ODE with time–varying coefficient through $S(t)$:

$$\dot{B}(t) + \underbrace{(1 - r_k)\frac{r_{\text{out}}}{S(t)}}_{=:a(t)} B(t) = r_{\text{in}}.$$

An integrating factor yields the closed form

$$B(t) = \left(\frac{S(t)}{S_0}\right)^{-\gamma} B_0 + \frac{r_{\text{in}}\,S_0}{\beta(1+\gamma)}\left[\frac{S(t)}{S_0} - \left(\frac{S(t)}{S_0}\right)^{-\gamma}\right], \qquad \gamma := \frac{(1-r_k)r_{\text{out}}}{\beta}, \quad B_0 := B(t_0), \tag{11}$$

valid when $\beta \neq 0$. The backing ratio $y(t) := B(t)/S(t)$ admits the compact expression

$$y(t) = \frac{r_{\text{in}}}{\frac{r_{\text{in}}}{\bar{P}} - r_k r_{\text{out}}} + \left(\frac{S(t)}{S_0}\right)^{-(1+\gamma)}\left(y_0 - \frac{r_{\text{in}}}{\frac{r_{\text{in}}}{\bar{P}} - r_k r_{\text{out}}}\right), \qquad y_0 := \frac{B_0}{S_0}. \tag{12}$$

If $\beta = 0$, then S is constant and B solves $\dot{B} + \alpha B = r_{\text{in}}$ with $\alpha = (1 - r_k)r_{\text{out}}/S_0$, yielding exponential convergence to the unique steady ratio stated below.

5.1 Steady states and asymptotic ratios

A steady state for (B, S) with constant rates and constant issuance price satisfies $\dot{S} = 0$ and $\dot{B} = 0$. The first condition implies $r^*_{\text{out}} = \frac{r_{\text{in}}}{\bar{P}}$; substituting into the second gives

$$\frac{B^*}{S^*} = \frac{r_{\text{in}}}{(1 - r_k)r^*_{\text{out}}} = \frac{\bar{P}}{1 - r_k}.$$

Combining with (4) yields the steady marginal floor:

$$P^{\text{floor},*} = (1 - \phi_{\text{tot}})\,\bar{P}, \tag{13}$$

which depends on the token fee but is *independent* of the cash–out tax r_k. This identity is a convenient design guideline: under balanced issuance and redemption activity within a stage, the floor tracks the contract issuance price up to the token fee multiplier.

When $\beta > 0$ (sustained net issuance), (12) shows that $y(t)$ converges to the limit

$$y^\infty = \frac{r_{\text{in}}}{\frac{r_{\text{in}}}{\bar{P}} - r_k r_{\text{out}}} \in \left(\frac{\bar{P}}{1 - r_k}, \infty\right),$$

and the floor tends to $(1 - \phi_{\text{tot}})(1 - r_k)\,y^\infty$. When $\beta < 0$ (sustained net redemptions), S decreases linearly and the formula remains valid up to the first time t such that $S(t) = 0$.

6 Design Implications and Interpretation

The continuous–time system clarifies how stage parameters and expected activity shape value accrual.

First, equation (8) decomposes floor growth into two mechanically distinct channels. The *issuance channel* increases the floor whenever the stage contract issuance price exceeds the current backing per token; this threshold depends on $\bar{P}$ and the state (B, S) but, importantly, does not depend on the split parameter σ_k, because the total minted supply per unit of base asset is $1/\bar{P}$ regardless of split allocation across recipients. The *redemption channel* increases the floor whenever there is positive redemption activity and a strictly positive cash–out tax r_k, because the tax retains value in the treasury while burning circulating tokens.

Second, when expected issuance and redemption rates balance within a stage, the steady floor is proportional to the contract issuance price as in (13). This provides a direct bridge from economic design targets (a desired floor trajectory) to the stage schedule of issuance prices, up to the multiplicative loss from token fees.

Third, the neutral line (9) characterizes the combinations of expected cash–in and cash–out activity that keep the floor locally stationary for the current state and issuance price. Crossing this line in the direction of larger activity generally accelerates floor growth.

Finally, scheduled auto–issuances create downward jumps in the floor because S increases discontinuously while B does not; this is immediate from (4) and the jump condition (7). Such events should therefore be used judiciously and, when possible, timed after periods of strong issuance or redemption–driven floor growth.

Conclusion

Modeling Revnet value flows as a continuous–time, rate–based system yields tractable and interpretable dynamics. The ODEs (5)–(6) conserve the core accounting identities, the floor derivative formula (8) isolates the precise conditions for appreciation, and the closed forms (10)–(12) provide immediate tools for analysis and calibration on any interval where the issuance price is constant. These results are sufficient to study stage design and to backtest trajectories using empirically estimated rate functions.

A Adding Loans as Aggregate Rates

Although the main text focuses on issuance and redemption in expectation, loans can be incorporated additively at the level of rates without changing the qualitative conclusions.

Let $\ell_L(t) \in \mathbb{R}_{\geq 0}$ denote the expected *gross* principal originated per unit time (base units/time), let $\rho_c(t) \in \mathbb{R}_{\geq 0}$ denote the expected collateral burned per unit time (tokens/time), let $\pi(t) \in \mathbb{R}_{\geq 0}$ denote expected principal repaid per unit time, and let $f_t(t) \in \mathbb{R}_{\geq 0}$ denote expected *time–fee* payments per unit time. Let $F_{\text{pre}} \in [0, 1)$ be the mean prepaid fee fraction retained by the same revnet at origination. On an interval where the contract issuance price is constant and loan fee payments to the same revnet are treated as ordinary pay–ins that mint at that price, the ODEs become

$$\dot{S}(t) = \frac{r_{\text{in}}(t)}{\bar{P}} - r_{\text{out}}(t) \;-\; \rho_c(t) \;+\; \frac{F_{\text{pre}}\, \ell_L(t)}{\bar{P}} \;+\; \frac{f_t(t)}{\bar{P}},$$

$$\dot{B}(t) = r_{\text{in}}(t) - (1 - r_k)\frac{B(t)}{S(t)}\, r_{\text{out}}(t) \;-\; (1 - F_{\text{pre}})\ell_L(t) \;+\; \pi(t) \;+\; f_t(t).$$

Substituting into (8), one finds that loan origination increases the floor provided the *effective* pricing of loans satisfies the standard solvency constraint (gross principal per token of collateral strictly less than B/S), while full principal repayment decreases the floor by reversing the principal effect; prepaid and time fees add to supply and to the treasury in a way analogous to ordinary pay–ins.